

Code: 011410, Hydraulics & open channel flow

Sem.

- i) - (b)
- ii) - (c)
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- iv) - (a)
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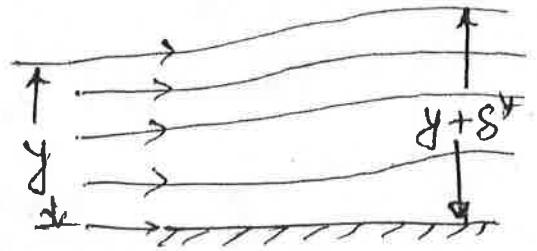
Subject:- Hydraulics & Open Channel Flow

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Solution of

Sets (I) / (II)

Q2. The displacement thickness, δ^* , may be defined as the distance measured perpendicular to the boundary by which the free stream is displaced on account of formation of boundary layer.



displacement thickness can be written as

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{u_{\infty}}\right) dy \quad \text{--- (1)}$$

Momentum Thickness, θ , may be defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for reduction in momentum of the flowing fluid on account of formation of B.L.

The momentum Thickness, θ , can be written as

$$\theta = \int_0^{\delta} \frac{u}{u_{\infty}} \left(1 - \frac{u}{u_{\infty}}\right) dy \quad \text{--- (2)}$$

Mathematical Analysis - Part 2
 Chapter 10

Exercise 10.1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function. Show that f is differentiable at a if and only if f is differentiable at a and $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.

Proof. Suppose f is differentiable at a . Then by definition, there exists a unique real number L such that $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = L$. We claim that $L = f'(a)$. To see this, note that if f is differentiable at a , then $f'(a)$ is defined as the limit $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$. Since this limit exists and is unique, it must be equal to L . Thus $f'(a) = L$.



Conversely, suppose f is differentiable at a and $f'(a) = L$. We want to show that f is differentiable at a . Let $\epsilon > 0$ be given. Since $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = L$, there exists $\delta > 0$ such that $0 < |h| < \delta$ implies $|\frac{f(a+h) - f(a)}{h} - L| < \frac{\epsilon}{2}$. Then $|\frac{f(a+h) - f(a)}{h} - L| < \frac{\epsilon}{2}$ implies $|\frac{f(a+h) - f(a)}{h} - L| < \frac{\epsilon}{2}$. Thus $|\frac{f(a+h) - f(a)}{h} - L| < \frac{\epsilon}{2}$.

$$\left| \frac{f(a+h) - f(a)}{h} - L \right| < \frac{\epsilon}{2}$$

Therefore, $|\frac{f(a+h) - f(a)}{h} - L| < \frac{\epsilon}{2}$ implies $|\frac{f(a+h) - f(a)}{h} - L| < \frac{\epsilon}{2}$. This shows that f is differentiable at a and $f'(a) = L$. Thus f is differentiable at a if and only if f is differentiable at a and $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.

$$\left| \frac{f(a+h) - f(a)}{h} - L \right| < \frac{\epsilon}{2}$$

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Model Answer

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Sets (I) / (II)

$$Re_L = \frac{UL}{\nu} = \frac{5 \times 4}{1.47 \times 10^{-5}} = 1.36 \times 10^6 > 5 \times 10^5$$

Flow is turbulent at rear side.

$$Re_x \frac{Ux}{\nu} = 5 \times 10^5 \Rightarrow x = \frac{5 \times 10^5 \times 1.47 \times 10^{-5}}{5} = 1.47 \text{ m}$$

Hence B.L. is laminar on 1.47 m length of the plate.

$$\delta = \frac{5 \times x}{\sqrt{Re_x}} = \frac{5 \times 1.47}{\sqrt{5 \times 10^5}} = 0.01039 \text{ m} \\ = 1.039 \text{ cm.}$$

$$c_f = \frac{0.664}{\sqrt{Re_x}} = 0.000939$$

$$\tau_w = c_f \frac{\rho U^2}{2} = 0.000939 \times \frac{1.208 \times 5^2}{2} \\ = 0.01418 \text{ N/m}^2$$

Force on 1.47m length (on both sides) will be

$$F = 2 \times (2 \times 1.47) c_f \frac{\rho U^2}{2}$$

$$c_f = \frac{1.328}{\sqrt{Re_x}} = 0.001878$$

$$F = 2 \times (2 \times 1.47) \times 0.001878 \times 1.208 \times \frac{5^2}{2}$$

$$F = 0.1667 \text{ N.}$$

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Ques 10

$$P_1 = \frac{1}{2} = \frac{2 \times 10^3}{1.12 \times 10^2} = 1.78 \times 10^1 \text{ Pa}$$

Flow is turbulent

$$P_2 = \frac{1}{2} = \frac{2 \times 10^3}{1.12 \times 10^2} = 1.78 \times 10^1 \text{ Pa}$$

There is a pressure loss in the pipe

$$\Delta P = \frac{2 \times 10^3}{1.12 \times 10^2} = 1.78 \times 10^1 \text{ Pa}$$

$$\Delta P = \frac{0.6 \times 10^3}{1.12 \times 10^2} = 5.36 \text{ Pa}$$

$$\Delta P = \frac{0.6 \times 10^3}{1.12 \times 10^2} = 5.36 \text{ Pa}$$

$$\Delta P = 0.6 \times 10^3 / 1.12 \times 10^2$$

Force on water (on water) will be

$$F = \rho \times V \times g = \frac{1}{2} \times \frac{1}{2} \times 9.81$$

$$F = \frac{1}{2} \times \frac{1}{2} \times 9.81$$

$$F = \frac{1}{2} \times \frac{1}{2} \times 9.81$$

$$F = 0.6 \times 10^3$$

Force

Force

Force

Force

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Sets (I) / (II)

Soln of Q3

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = B^2 y_c^3$$

$$B = \sqrt{\frac{Q^2}{g}} y_c^{-3/2}; \quad P = B + 2y = \sqrt{\frac{Q^2}{g}} y_c^{-3/2} + 2y$$

$$\text{For min } P, \quad \frac{dP}{dy} = \sqrt{\frac{Q^2}{g}} \left(-\frac{3}{2} y^{-5/2} \right) + 2 = 0$$

$$y^{+5/2} = \frac{3}{4} \sqrt{\frac{Q^2}{g}} \Rightarrow \sqrt{\frac{Q^2}{g}} = \frac{4}{3} y^{5/2} \Rightarrow \frac{Q^2}{g} = \frac{16}{9} y^{10/2}$$

$$B^2 = \frac{16}{9} y^2; \quad \text{Hence } B = \frac{4}{3} y$$

$$\begin{aligned} E_c &= y_c + \frac{V_c^2}{2g} = \frac{3B}{4} + \frac{27}{64} B^5 \left(\frac{1}{2} + \frac{16}{9} \frac{1}{B^4} \right) \\ &= \left(\frac{3}{4} + \frac{3}{8} \right) B = \frac{9}{8} B \end{aligned}$$

$$\boxed{B = \frac{8}{9} E_c}$$

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Solⁿ of Q4

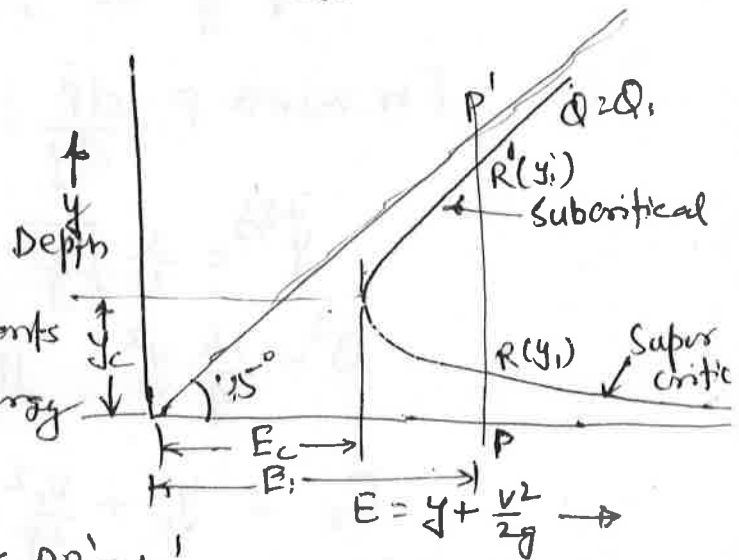
Sets (X/II)

$$E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2}$$

For a channel of known geometry, $E = f(y, Q)$

Now, Keeping $Q = \text{const.} = Q_1 \Rightarrow E = f(y)$

By drawing a curve betⁿ E vs y as show in figure.



The ordinate PP' represents the condition for a sp. energy E .

The depth of flow can be either $PR = y_1$ or $PP' = y_1'$

These two depths y_1 and y_1' are known as alternate depths. The intercept $P'R'$ or $P'R$ represents the velocity head.

For given Q , as the sp. energy is increased the difference betⁿ two depths increases.

On other hand, if E is decreased, the difference will decrease at $E = E_c$, the two depth will merge with each other. This is the minimum sp. energy and the critical point for flow, so it is known as critical flow cond.

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Solⁿ of Q.4 Continue

Set (A)/(II)

$$E = E_c = 1.2 \text{ m}$$

$$y_c = \frac{2}{3} E_c = 0.8 \text{ m}$$

$$q_m = (g y_c^3)^{1/2} = (9.81 \times (0.8)^3)^{1/2} = 2.241 \text{ m}^3/\text{s}/\text{m}$$

$$Q \approx Q_B = 2.241 \times 2.0 = 4.482 \text{ m}^3/\text{s}$$

$$R = \frac{2 \times 0.8}{2 + 1.6} = 0.4444 \text{ m}$$

$$\text{Conveyance } K = \frac{1}{n} A R^{2/3} = \frac{1}{0.015} 1.6 \times (0.4444)^{2/3} = 62.12 \text{ m}^3/\text{s}$$

$$S_0 = \left(\frac{Q}{K} \right)^2 = \left(\frac{4.482}{62.12} \right)^2 = 5.206 \times 10^{-3}$$

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Sets (I)/(II)

Solⁿ of Q5

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}}$$

$\frac{dy}{dx} \rightarrow$ Water surface profile variation with x to x .

$S_0 \rightarrow$ Bottom Slope = $\frac{dz}{dx}$

$S_f \rightarrow$ Energy Slope = $\frac{dH}{dx}$

$\frac{Q^2 T}{g A^3} \rightarrow F_r^2 = \text{Froude No.}$

For wide rectangular channel $q^2 = \frac{1}{n^2} y_0^{10/3}$

At critical depth $\frac{q^2}{g} = y_c^3$

At critical slope, $S_0 = S_c$ and $y_0 = y_c$

$$g y_c^3 = \frac{1}{n^2} y_c^{10/3}$$

$$S_c = \frac{g n^2}{y_c^{1/3}} = \frac{g n^2 g^{1/9}}{q^{2/9}} =$$

$$\frac{g^{10/9} \cdot n^2}{q^{2/9}} = S_c$$

If $S_0 > S_c$, the flow is supercritical, slope is steep.

If $S_0 < S_c$, the flow is subcritical, slope is mild.

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Soln. of Q6

Set (D)/(11)

$$A = 4.0y, V = \frac{15}{A}, P = (4 + 2y), E = y + \frac{V^2}{2g}$$

$$S_f = \frac{n^2 V^2}{R^{4/3}} = \frac{(0.016)^2 V^2}{R^{4/3}}$$

$$\phi = \frac{Qn}{\sqrt{S_0} B^{8/3}} = \frac{15 \times 0.016}{\sqrt{0.0009} \times (4)^{2.67}} = 0.198425$$

Corresponding value of $\frac{y_0}{B} = 0.5$ and $y_0 = 2.0$ m.

$$y_c = \left[\frac{(15.0/4)^2}{9.81} \right]^{1/3} = 1.128 \text{ m}$$

If $y_1 = 2.6$ m, $y_1 > y_0 > y_c$ Hence Water Surface Profile is M₁.

The depth down stream of A would be higher than 2.6 m.

y (m)	A (m ²)	R (m)	V (m/s)	E (m)	DE (m)	S _f	\bar{S}_f	$S_0 - \bar{S}_f$	Δx	x
2.6	10	1.13	1.44	2.71		0.0004527				
2.8	11	1.16	1.36	2.84	0.1387	0.0003915	0.000422	0.000478	290	0
2.88	12	1.18	1.30	2.97	0.121633	0.0003479	0.00037	0.00053	229	290
										520

By interpolation depth at section B 500m from A is $2.80 + [(0.08/230) \times 210] = 2.873$ m.

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Soln of Q7.

Sets (I)/(II)

By momentum Eqn $P_1 + M_1 = P_2 + M_2$

$$\rho g \frac{m}{3} y_1^3 + \frac{\rho Q^2}{m y_1^2} = \rho g \frac{m}{3} y_2^3 + \frac{\rho Q^2}{m y_2^2}$$

$$\frac{Q^2}{m^2} \left(\frac{1}{y_1^2} - \frac{1}{y_2^2} \right) = \frac{g}{3} (y_2^3 - y_1^3)$$

Here $m = 1$.

$$Q^2 \left[\frac{1}{(0.6)^2} - \frac{1}{(1.2)^2} \right] = \frac{9.81}{3} (1.2^3 - (0.6)^3)$$

$$2.0833 Q^2 = 4.94424$$

$$Q = 1.541 \text{ m}^3/\text{s}$$

$$F_1^2 = \frac{2Q^2}{g^2 m^2 y_1^5} = \frac{2 \times (1.541)^2}{(9.81)^2 (0.6)^5} = 6.222$$

$$F_1 = 2.494$$

Similarly $F_2^2 = \frac{2Q^2}{g^2 m^2 y_2^5} = 0.1964$

$$F_2 = 0.443$$

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Solution of Q 8

Sets (I) / (II)

$$q_1 = 1.5 \text{ m}^3/\text{s}/\text{m} = V_1 y_1$$

$$y_1 = 0.75 \text{ m}$$

$$V_1 = \frac{1.5}{0.75} = 2.0 \text{ m/s}$$

$$q_2 = V_2 y_2 = 1.33 \times 1.5 = 1.995 \text{ m}^3/\text{s}/\text{m} \quad (2)$$

$$y_2 (V_w - V_2) = y_1 (V_w - V_1)$$

$$y_2 V_w - 1.995 = 0.75 V_w - 1.5$$

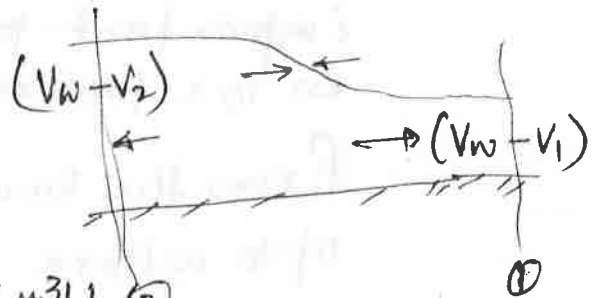
$$V_w (y_2 - 0.75) = 0.495 \quad \text{--- (1)}$$

Now, we know,

$$(V_w - V_1)^2 = \frac{g}{2} \left[\frac{y_2}{y_1} (y_1 + y_2) \right]$$

$$\left[\frac{0.495}{y_2 - 0.75} - 2 \right]^2 = \frac{9.81}{2} \cdot \frac{y_2}{0.75} (0.75 + y_2)$$

By Trial & Error, $y_2 = 0.85 \text{ m}$ and $V_w = 4.95 \text{ m/s}$.



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Solution of Q. 9.

Sets (I) / (II)

(i) Boundary Layer Thickness (BLT) is very important parameter that must be determined in B.L. problem. It is defined as the distance from the boundary to the point vertically upwards upto where the velocity differs by 1% from the free stream velocity. The condition for B.L.T, δ ;

$$y = \delta \text{ for } u = 0.99u_{\infty}$$

(ii) Specific force

Steady-state momentum equation takes a simple form if the tangential force and body force are both zero.

$$F_1 + M_1 = F_2 + M_2 = \text{Const} = P_3$$

P_3 is known as specific force and represents the sum of the pressure force and momentum flux per unit weight of the fluid at a section.

Specific force is function of depth of flow, channel geometry and discharge.

$$P_3 = \frac{P+M}{\gamma} = A\bar{y} + \frac{Q^2}{gA} = \text{Constant}$$

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Solution of Q.9 Sets (I) / (II)

(iii.) Factors Affecting 'n'

The Manning's n is essentially a coefficient representing the integrated effects of a large number of factors contributing to the energy loss in a reach.

These are:

- Surface roughness
- Vegetation
- Cross-section irregularity
- Irregular alignment of channel of the surface.

(2V.) Hydraulically efficient channel is one which, for a given slope, roughness coefficient and area of flow, carries the maximum flow rate. The conveyance of a section of a given area increases with decrease in its perimeter. Hence, a channel section having minimum perimeter for a given area of flow provides the maximum value of conveyance and hence maximum discharge.

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Sets (I) / (II)

Hence a channel with the slope, roughness and area of flow fixed, a minimum perimeter section will represent the hydraulically efficient section.

- (V) Unsteady flow occurs in an open channel when the discharge or depth or both vary with respect to time at a section. These changes may be due to natural causes, planned action or accidental happening. Depending upon the curvature of water surface, this can be classified as gradually varied unsteady flow and rapidly varied unsteady flow. Unsteady flow causes are heavy rainfall in a catchment, snow melt which give rise to floods in rivers, stream etc. operation of control gates in hydraulic structures opening or closing of gate of turbine etc.

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